

ANOMALY IN THE CHARGE RADII OF Pb ISOTOPES

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Abstract

The anomalous behaviour of the charge radii of the isotopic chain of Pb nuclei has been studied in the relativistic mean field theory. It has been shown that the relativistic mean field provides an excellent description of the anomalous kink in the isotopic shifts about ^{208}Pb . This contrasts strongly from the Skyrme mean field, where the density-dependent Skyrme forces fail to reproduce the observed trend in the empirical data on the charge radii. The results have been discussed in the perspective of differences in the ansätze of the relativistic and the Skyrme mean-field theories.

The relativistic mean field (RMF) theory [1,2] has of late enjoyed enormous success in providing an appropriate description of the ground-state properties of nuclei. It has also been received as a viable alternative to the phenomenological density dependent Skyrme forces [3]. The built-in spin-orbit interaction of the Dirac description of nucleons renders an attractive feature. On the other hand, the spin-orbit interaction is added only phenomenologically in the non-relativistic Skyrme forces. In the RMF theory the saturation and the density dependence of the nuclear interaction is provided by a balance between large attractive scalar σ meson and large repulsive vector ω meson. The asymmetry component is provided by the isovector ρ meson. The nuclear interaction is hence generated by the exchange of various mesons between nucleons. It is surmised that the structure of the force in the RMF theory and its density dependence thus differ from that of Skyrme forces, where in the latter a certain density dependence is assumed at the outset.

There have been attempts [2] to understand the similarities and differences between the RMF theory and the Skyrme approaches. Although similarities are found, basic differences are, however, not yet understood. One of the differences in these approaches is the acceptability of the different values of the effective nucleon mass. It has been shown [4] that for a satisfactory description of the spin-orbit splitting in nuclei, a lower value of the effective mass around 0.55-0.60 is required in the RMF theory. This is in contrast to the effective mass about 0.80 obtained from the Skyrme forces. A larger value of the effective mass in the RMF theory would produce a very small spin-orbit splitting, that would be incompatible with the observed magnitude. This effect was also demonstrated [5] clearly in other variants of the ansatz for scalar coupling, whereby the couplings leading to higher effective mass showed an inadequacy in describing the spin-orbit splitting. This difference in the requirement of the effective masses in the Skyrme and RMF approaches is incomprehensible. It has, however, been pointed out [6] that the two effective masses should not be compared, as the two values do not correspond to the same property.

The charge radii of Pb isotopes and their isotope shifts have been a matter of detailed discussion [7] within the framework of the Skyrme forces. The

isotopic chain of Pb nuclei exhibit a well known kink in the behaviour of the empirical isotope shifts [8]. This implies that with a gradual addition of neutrons, the charge radii of nuclei heavier than ^{208}Pb do not show a trend similar to that of lighter isotopes. The Skyrme forces do succeed in describing the isotopes shifts and thus charge radii of nuclei on the lighter side of ^{208}Pb , where a density-dependent pairing force is required to be assumed. All the Skyrme forces, however, fail to reproduce the isotope shifts of the heavier counterparts, as has been discussed in detail in Ref. 7. This indicates that remaining within the Skyrme ansatz, the mean field is not able to describe heavier isotopes of Pb. In the present work, we investigate this long standing problem within the framework of the RMF theory.

As stated above, the RMF theory represents a framework that provides an excellent description of the ground-state properties of nuclei [9]. The ansatz of the interaction in the RMF theory is based upon Lagrangian of the form [1]:

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\rlap{\not{\partial}} - M)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \\ & \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\rlap{\not{\omega}}\psi - g_\rho\bar{\psi}\vec{\rho}\vec{\tau}\psi - e\bar{\psi}\rlap{\not{A}}\psi.\end{aligned}\tag{1}$$

where the Dirac nucleon interacts with the σ and ω meson fields. The ρ meson generates the isovector component of the force. The nonlinear $\sigma\omega\rho$ model which we employ, has a nonlinear scalar self-interaction of the σ mesons as given by

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4,\tag{2}$$

where g_2 and g_3 are the non-linear parameters. Details on the RMF theory have been discussed in Ref. 2. There have been a few parameter sets [2] in the RMF theory, which have been used extensively to obtain the ground-state properties, the sets NL1 [10] and NL2 [11] being among the few. Both the above forces provide neutron radii and thus neutron skin thickness [12] of neutron-rich nuclei much larger than the empirical values.

Recently, the ground-state properties of nuclei have been investigated [9] in the non-linear $\sigma\omega\rho$ model. It has been noted that indeed a stronger ρ meson coupling and hence a very large asymmetry energy of the above forces gives a larger neutron skin thickness of neutron-rich nuclei. Consequently, a new force NL-SH has been obtained, where the above defect of the earlier forces has been removed. It has also been shown [9] that this force describes the ground-state binding energies, charge and neutron radii of spherical as well as of deformed nuclei very far off the stability line very well. We employ the above force to obtain the ground-state properties of the isotopic chain of Pb nuclei. The parameters of the force NL-SH are the following:

$$M = 939.0 \text{ MeV}; m_\sigma = 526.059 \text{ MeV}; m_\omega = 783.0 \text{ MeV}; m_\rho = 763.0 \text{ MeV}; \\ g_\sigma = 10.444; g_\omega = 12.945; g_\rho = 4.383; g_2 = -6.9099; g_3 = -15.8337.$$

The calculations have been performed within the Hartree approximation. Although most of the Pb isotopes close to ^{208}Pb are spherical, an axially symmetric configuration has been assumed and Hartree minimization has been performed. The method of the oscillator expansion [13] has been employed, whereby both the fermionic as well as bosonic wavefunctions have been expanded in $N = 12$ shells. We have considered all the even mass Pb isotopes from $A = 190$ to 214 . For convergence reasons, $N = 14$ Fermionic shells have also been considered. It has been found that the difference between the $N=12$ and $N=14$ calculations is miniscule. Thus, we present only the results obtained with $N=12$. For all the open-shell nuclei pairing has been included within the BCS formalism with the pairing gap obtained from the particle separation energies of the neighbouring nuclei. The quadrupole deformations obtained from the convergence are very small and correspond essentially to spherical configuration for all the nuclei considered here. Figure 1 shows

the binding energy per nucleon of Pb isotopes obtained in the relativistic Hartree approximation. The empirical binding energies are also shown for comparison. It can be seen clearly that the binding energies of all isotopes are reproduced well for the set NL-SH. The deviations are at most 0.1%, and in many cases better. The overall agreement with the empirical data is remarkable. The calculated energies for the set NL1 show agreement with the empirical binding energies only in the vicinity of ^{208}Pb . This is expected as the force NL1 was obtained [10] by fitting binding energies including those

of ^{208}Pb . In going to the lighter side of the isotopic chain, the binding energies obtained with NL1 show a systematic deviation from the empirical data. The difference between the calculated and the empirical binding energies increases as the neutron excess decreases. This discrepancy is inevitably due to the large asymmetry energy of about 44 MeV for NL1. The problem has been rectified [9] in the force NL-SH.

The rms charge radii have been obtained as usual by folding the proton form factor with the proton density distribution. The charge radii of Pb isotopes obtained for NL-SH and NL1 are shown in Fig. 2. The empirical charge radii for Pb isotopes, as shown in this figure, have been obtained from the measured isotope shifts [8] employing the empirical [14] charge radius of ^{208}Pb as 5.503 fm. The charge radii from NL-SH follow closely the empirical ones implying that the experimental data is well described by NL-SH. The force NL1, on the other hand, overestimates the charge radii of Pb isotopes and especially those of the lighter neutron-deficient ones.

The calculated charge radii have been used to obtain the isotope shifts. The nucleus ^{208}Pb has been taken as the reference point. As in ref. 7, the isotope shifts ($\Delta r_c^2 = r_c^2(A) - r_c^2(208)$) have been modified by subtracting an equivalent of the liquid-drop difference ($\Delta r_{LD}^2 = r_{LD}^2(A) - r_{LD}^2(208)$) obtained from $r_{LD}^2(A) = \frac{3}{5}r_0^2 A^{2/3}$, where $r_0 = 1.2$ fm. Fig. 3 shows the calculated value of the modified isotope shifts ($\Delta r_c^2 - \Delta r_{LD}^2$) for Pb nuclei. All the data have been presented in the same fashion. The empirical values are from the precision data obtained from the atomic beam laser spectroscopy [8]. The empirical data exhibit a conspicuous kink about ^{208}Pb . The figure also shows the theoretically obtained isotope shifts for the two forces NL-SH and NL1 and compare them with those from Skyrme interaction SkM*. Remarkably, isotope shifts from NL-SH reproduce the kink very well. Only below $A = 198$ the theoretical isotope shifts show a systematic divergence from the empirical data. This behaviour is well below the kink and may be attributed to the transitional behaviour of nuclei in the light Pb isotopes. This region of mass number is prone to such effects. The force NL1, on the other hand, shows a reasonable kink on the higher side of ^{208}Pb . On the lower side, NL1 shows a slight divergence from the data and the slope of the theoretical values is also different from that of the empirical data. It may be noted that due to

inaccurate description of the other ground-state properties and a very large asymmetry energy, NL1 is not expected to provide a proper behaviour of the isotope shifts. In Fig. 3 we also show for comparison isotope shifts for SkM* as taken from ref. 7. The data points on ^{194}Pb and ^{214}Pb are representative of the behaviour of SkM* on the isotope shifts. On the lighter side, SkM* shows a behaviour similar to NL-SH. It, however, shows an almost linear function with mass number and consequently displays a conspicuous divergence from the empirical data on heavier side.

The kink in the experimental data implies that some changes in placement of extra neutrons in a new shell take place, which bring about this kink. Attempts to reproduce this kink within the density-dependent Skyrme forces have been discussed in detail in Ref. 7. As shown in fig. 2 of Ref. 7, within the Hartree-Fock mean field, all the Skyrme forces e.g. SkM*, Ska and SIII show a strong deviation from the empirical data in the isotope shifts for nuclei heavier than ^{208}Pb . The lighter Pb isotopes could, however, be described by SkM* and SGII. The binding energies, on the other hand, show a behaviour opposite to that of isotope shifts. Whereas SkM* reproduces the binding energies of isotopes including and heavier than ^{208}Pb , it shows a systematic divergence from empirical binding energies for lighter isotopes, the difference being accentuated on going to the neutron-deficient side. The other two Skyrme forces show disagreements with the empirical binding energies on both the sides. Taking all possible corrections beyond the mean field into account does not resolve the discrepancy. It has been demonstrated unambiguously in Ref. 7 that Skyrme forces face a tremendous problem in describing the binding energies and isotopes shifts of nuclei away from ^{208}Pb . It has also been concluded [7] that the compressibility of the nuclear matter does not have a decisive effect on the isotope shifts of nuclei in contrast to what was stated earlier in ref. [15]. It is worth noting that most of the Skyrme forces have been fitted to the binding energies and charge radii of spherical nuclei from ^{16}O to ^{208}Pb , with almost comparable surface and asymmetry energies, but with a strong difference in the incompressibility and the equation of state (EOS). It has been shown [16] that even with a very broad variation in the EOS, one can construct any number of Skyrme interactions that could describe the ground-state properties of some spherical

nuclei from ^{16}O to ^{208}Pb . The forces SkM*, SGII and SIII manifest a similar scenario as discussed in Ref. [16]. The failure of these well-known Skyrme forces to reproduce the empirical data on Pb isotopes raises an important question as to whether the Skyrme ansatz could be extrapolated to describe nuclei far off the stability notwithstanding the problem associated with the Skyrme forces even on the stability line.

The ability to reproduce the kink by the RMF theory and the failure of the present Skyrme interactions not to be able to do so, raises some important issues. The basic shell effects that are inherent in the structure of nuclei over the periodic table provide a clue as to how effectively various theories are able to take these effects into account. The kink in the empirical data on charge radii of Pb isotopes belongs to these questions. Evidently, the RMF theory succeeds in accommodating these shell effects which run across the shell-closure. The shell effects across the magic numbers play a significant role in astrophysical r-processes.

The behaviour of the Skyrme mean-field approach and of the RMF theory towards isotope shifts indicates an important difference between the two approaches. This difference is not easy to explain. However, examining the two approaches we feel that the basic density dependence of the two interactions and the difference in the saturation mechanism of the two methods could be at the origin of the difference in the isotope shifts. Another important aspect that is different is the spin-orbit interaction. Whereas in the RMF theory, the spin-orbit interaction originates from the coupling of σ and ω mesons to Dirac nucleon, the spin-orbit term in the Skyrme approach is added phenomenologically. The spin-orbit splitting is responsible for putting different orbitals in space and thus determining the structure of a nucleus. Thus, a difference in the spin-orbit splitting in the two methods would contribute to the difference in the isotope shifts of the two approaches. Further work in this direction is in progress. In conclusion, we have examined the behaviour of the RMF theory in describing the anomalous isotope shifts in Pb nuclei. It has been shown that the RMF theory describes the isotope shifts successfully in marked contrast to the Skyrme mean field approach. This difference has unveiled some issues underlying the applicability of the two approaches in nuclear structure. It is expected that these differences will

have broader implications on the properties of neutron-rich nuclei far from the stability line, and in particular, for nuclei close to neutron-drip line, with consequences on the EOS of neutron matter and neutron-star structure.

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Figure Captions.

Fig. 1 The binding energies of Pb isotopes obtained in the relativistic Hartree approximation with the forces NL1 and NL-SH. The empirical values (expt.) are also shown for comparison.

Fig. 2 The charge radii of Pb isotopes obtained with NL1 and NL-SH. The empirical values (expt.) from laser spectroscopic measurements [9] follow closely the charge radii from NL-SH.

Fig. 3. The isotope shifts of Pb nuclei obtained with NL1 and NL-SH. The empirical values [9] along with the values from NL-SH exhibit a conspicuous kink in the isotopes shifts about ^{208}Pb . The SkM* values [7] show a large deviation from the empirical data for heavier nuclei.

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